

Paper Reference 9FM0/02
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced

PAPER 2:

Core Pure Mathematics 2

Monday 5 June 2023 – Afternoon

Time: 1 hour 30 minutes

YOU MUST HAVE

**Mathematical Formulae and Statistical
Tables (Green), calculator**

YOU WILL BE GIVEN

Diagram Booklet

Answer Booklet

Y72795A

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or on the separate diagrams – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

Turn over

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

There are 9 questions in this question paper.

The total mark for this paper is 75.

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. Refer to the diagram for Question 1 in the Diagram Booklet.

The diagram shows a sketch of the curve with polar equation

$$r = 2\sqrt{\sinh \theta + \cosh \theta} \quad 0 \leq \theta \leq \pi$$

The region **R**, shown shaded in the diagram, is bounded by the initial line, the curve and the line with equation

$$\theta = \pi$$

(continued on the next page)

1. continued.

Use algebraic integration to determine the exact area of R giving your answer in the form $pe^q - r$ where p , q and r are real numbers to be found.

(Total for Question 1 is 4 marks)

2. (a) Write down the Maclaurin series of e^x , in ascending power of x , up to and including the term in x^3

(1 mark)

(continued on the next page)

2. continued.

**(b) Hence, without differentiating,
determine the Maclaurin series of**

$$\mathbf{e^{(e^x - 1)}}$$

**in ascending powers of x , up
to and including the term in
 x^3 , giving each coefficient in
simplest form.**

(5 marks)

(Total for Question 2 is 6 marks)

Turn over

3. $\mathbf{M} = \begin{pmatrix} -2 & 5 \\ 6 & k \end{pmatrix}$

where k is a constant.

Given that

$$\mathbf{M}^2 + 11\mathbf{M} = a\mathbf{I}$$

where a is a constant and $\mathbf{I}$ is the
 2×2 identity matrix,

(continued on the next page)

Turn over

3. continued.

(a) (i) determine the value of a

(ii) show that

$$\mathbf{k = -9}$$

(3 marks)

**(b) Determine the equations
of the invariant lines of the
transformation represented by M**

(6 marks)

(continued on the next page)

Turn over

3. continued.

(c) State which, if any, of the lines identified in (b) consist of fixed points, giving a reason for your answer.

(1 mark)

(Total for Question 3 is 10 marks)

Turn over

4. (a) Sketch the polar curve **C**, with equation

$$r = 3 + \sqrt{5} \cos \theta \quad 0 \leq \theta \leq 2\pi$$

On your sketch clearly label the pole, the initial line and the value of r at the point where the curve intersects the initial line.
(2 marks)

(continued on the next page)

4. continued.

The tangent to **C** at the point **A**,
where

$0 < \theta < \frac{\pi}{2}$, is parallel to the initial
line.

(b) Use calculus to show that at **A**

$$\cos \theta = \frac{1}{\sqrt{5}}$$

(4 marks)

(continued on the next page)

Turn over

4. continued.

- (c) Hence determine the value of
r at A
(1 mark)**

(Total for Question 4 is 7 marks)

Turn over

5. The points representing the complex numbers

$$z_1 = 35 - 25i \text{ and}$$

$z_2 = -29 + 39i$ are opposite vertices of a regular hexagon, H , in the complex plane.

The centre of H represents the complex number α

(a) Show that

$$\alpha = 3 + 7i$$

(2 marks)

(continued on the next page)

Turn over

5. continued.

Given that

$$\beta = \frac{1+i}{64}$$

(b) show that

$$\beta(z_1 - \alpha) = 1$$

(2 marks)

(continued on the next page)

Turn over

5. continued.

The vertices of H are given by the roots of the equation

$$(\beta(z - \alpha))^6 = 1$$

(c) (i) Write down the roots of the equation

$$w^6 = 1 \text{ in the form } re^{i\theta}$$

(1 mark)

(continued on the next page)

Turn over

5. (c) continued.

- (ii) Hence, or otherwise,
determine the position of
the other four vertices of H ,
giving your answers as
complex numbers in
Cartesian form.
(4 marks)**

(Total for Question 5 is 9 marks)

Turn over

6. Given that

$$y = e^{2x} \sinh x$$

prove by induction that for $n \in \mathbb{N}$

$$\frac{d^n y}{dx^n} =$$

$$e^{2x} \left(\frac{3^n + 1}{2} \sinh x + \frac{3^n - 1}{2} \cosh x \right)$$

(Total for Question 6 is 6 marks)

Turn over

- 7. In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(continued on the next page)

7. continued.

Look at the diagram for Question 7 in the Diagram Booklet.

John picked 100 berries from a plant.

The largest berry picked was approximately 2.8 cm long.

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Turn over

7. continued.

**The shape of this berry is modelled
by rotating the curve with equation**

$$16x^2 + 3y^2 - y \cos\left(\frac{5}{2}y\right) = 6 \quad x \geq 0$$

**shown in the diagram, about the
y-axis through 2π radians, where
the units are cm**

(continued on the next page)

Turn over

7. continued.

Given that the y intercepts of the curve are -1.545 and 1.257 to four significant figures,

- (a) use algebraic integration to determine, according to the model, the volume of this berry.
(6 marks)**

(continued on the next page)

Turn over

7. continued.

Given that the 100 berries John picked were then squeezed for juice,

(b) use your answer to part (a) to decide whether, in reality, there is likely to be enough juice to fill a 200 cm^3 cup, giving a reason for your answer.

(2 marks)

(Total for Question 7 is 8 marks)

Turn over

8. Given that a cubic equation has three distinct roots that all lie on the same straight line in the complex plane,

(a) describe the possible lines the roots can lie on.

(2 marks)

$$f(z) = 8z^3 + bz^2 + cz + d$$

where b , c and d are real constants.

(continued on the next page)

Turn over

8. continued.

The roots of $f(z)$ are distinct and lie on a straight line in the complex plane.

Given that one of the roots is

$$\frac{3}{2} + \frac{3}{2}i$$

**(b) state the other two roots of $f(z)$
(1 mark)**

(continued on the next page)

Turn over

8. continued.

$$g(z) = z^3 + Pz^2 + Qz + 12$$

where **P** and **Q** are real constants,
has **3** distinct roots.

The roots of **g(z)** lie on a different
straight line in the complex plane
than the roots of **f(z)**

(continued on the next page)

8. continued.

Given that

- **$f(z)$ and $g(z)$ have one root in common**
- **one of the roots of $g(z)$ is -4**

**(c) (i) write down the value of the common root,
(1 mark)**

**(ii) determine the value of the other root of $g(z)$
(3 marks)**

(continued on the next page)

Turn over

8. continued.

(d) Hence solve the equation

$$\mathbf{f(z) = g(z)}$$

(4 marks)

(Total for Question 8 is 11 marks)

Turn over

- 9. A patient is treated by administering an antibiotic intravenously at a constant rate for some time.**

Initially there is none of the antibiotic in the patient.

(continued on the next page)

9. continued.

At time t minutes after treatment began

- the concentration of the antibiotic in the blood of the patient is x mg/ml**
- the concentration of the antibiotic in the tissue of the patient is y mg/ml**

(continued on the next page)

Turn over

9. continued.

The concentration of antibiotic in the patient is modelled by the equations

$$\frac{dx}{dt} = 0.025y - 0.045x + 2$$

$$\frac{dy}{dt} = 0.032x - 0.025y$$

(continued on the next page)

Turn over

9. continued.

(a) Show that

$$40\,000 \frac{d^2y}{dt^2} + 2800 \frac{dy}{dt} + 13y = 2560$$

(3 marks)

(b) Determine, according to the model, a general solution for the concentration of the antibiotic in the patient's tissue at time t minutes after treatment began.

(5 marks)

(continued on the next page)

Turn over

9. continued.

- (c) Hence determine a particular solution for the concentration of the antibiotic in the tissue at time t minutes after treatment began.
(4 marks)**

(continued on the next page)

Turn over

9. continued.

To be effective for the patient the concentration of antibiotic in the tissue must eventually reach a level between 185 mg/ml and 200 mg/ml

- (d) Determine whether the rate of administration of the antibiotic is effective for the patient, giving a reason for your answer.**
- (2 marks)**

(Total for Question 9 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
